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PARAMETER DESIGN OF HEAT SINK: MULTIPLE TRADE-OFFS

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ABSTRACT

This paper is to develop a mathematical model, to optimize and to evaluate a heat sink on chip in Electronic Printed Board Assembly. The model emphasizes Thermo-Mechanical Behavior considering cost, heat and geometrical aspects. An optimization model has been developed that characterizes a heat sink at the parameter design stage. The model, which is a multi objective multi constraint nature, is formulated as a Compromise DSP format. A group of scenarios in one or two priority levels of the goals has been investigated.

NOMENCLATURE

A_b	area of exposed base
A_c	cross section of a fin
C_p	constant pressure specific heat
C_t	target value for cost
d_i	deviation variable
D_h	hydraulic diameter of fluid flow channel
f(di)	deviation function
$g_i(X)$	system constraint function
h	heat transfer coefficient
h_l	heat loss limit
k	thermal conductivity of sink material
k_f	thermal conductivity of fluid
$\dot{m}$	total mass flow rate of coolant trough channels
m	number of system goals
n	number of system variables
p+q	number of system constraints
p	equality constraints
P	fin perimeter
q	inequality constraints
Q	power of chip
R	equivalent thermal resistance
R_b	thermal resistance of exposed base
R_f	thermal resistance of fin
R_g	target value for thermal resistance
Re	Reynolds number based on hydraulic diameter
W_i	weight for the Archimedean case
X_j	design variable
Z	deviation function
ΔT	temperature difference between base and coolant
η	fin efficiency
ν	kinematic viscosity of fluid

INTRODUCTION

Electronic Packaging Design is a very broad and complex area which requires a multi-disciplinary approach from design stage to manufacturing. Heat is generated by many electronic components in electronic assemblies. Chips dissipate considerable amount of heat. For example, a 5 mm X 5 mm chip creates 10W heat.
[S . Oktay, 1986] Also, on-off cycle of an electronic product creates high temperature variations. Heat generated inside the Electronic Package can be harmful to the components and to the Printing Wiring Board itself. Generated heat must be removed. Heat removal from the electronic system becomes more important as chip power increases. One of the most common methods for heat removal is forced convection of air through

heat generators. Since chips create considerable amounts of heat, heat sinks are located at the top of the capsule to remove er design stage in the context of design optimization. A multi goal, multi constraint model for a heat sink has been established and solved. Instead of focusing on one aspect of design in the optimization, other significant factors that affect the design should also be considered by a multiple goal approach.

PROCEDURE : MULTIPLE TRADE-OFFS

Design engineers always face trade-offs during their design process. Facilitating the multiple trade-offs of an engineering system depends on circumstances, the model, and available tools. The approach taken here is to model a system or component, in this case a heat sink, to find the satisfying parameters considering main factors affecting the system. In other words, an optimization model, which is multi objective in nature, is applied with the same or different priority levels due to scenarios created by the designer. A Compromise Decision Support Problem(DSP) [Mistree, 1990] is used in this process as a tool for achieving our aim.

"Compromise DSP is a hybrid formulation in that it incorporates concepts from both traditional mathematical programming and goal programming and makes use of some new ones. The Compromise DSP is stated as follows

Given

An alternative that is to be improved through modification
Assumption used to model the domain of interest
The system parameters; and
All other relevant information

Find

The values of the independent system variables

X_j $\qquad$ $j = 1,\ldots\ldots,n$

The values of *deviation variables* (They indicate the extent to which the goals are achieved.)

d_i^- **,** d_i^+ $\quad$ $i=1,\ldots\ldots,m$

Satisfy

The *system constraints* that must be satisfied for the solution to be feasible.

$\boxed{g_i(x) = 0}$ $\qquad$ $i=1,\ldots\ldots,p$

$\boxed{g_i(x) \le 0}$ $\qquad$ $i=p+1,\ldots\ldots,p+q$

The system goals that must achieve a specified target value as far as possible.

$\boxed{A_i(X) +} \; d_i^- - d_i^+ = G_i$ $\qquad$ $i=1,\ldots\ldots,m$

The lower and upper bounds on the system

$X_j^{min} \le X_j \le X_j^{max}$ $\qquad$ $j = 1,\ldots\ldots,n$

$d_i^-, d_i^+ \ge 0$ and $d_i^-, d_i^+ \ge 0$

Minimize

heat. Thus, this paper is based on emphasizing how to deal with multiple trade-offs during paramet

The *deviation function,* which is a measure of the deviation of the system performance from that implied by the set of goals and their associated priority levels or relative weights.

Case a: Pre-emptive (lexicographic minimum)

$$Z = [f_1(d_i^-, d_i^+), \ldots, f_k(d_i^-, d_i^+)]$$

Case b: Archimedean

$$Z = \sum_{i=1}^{m} W_i(d_i^- + d_i^+), \qquad Z = \sum_{i=1}^{m} W_i = 1, \; W_i \ge 0\,''$$

An extensive overview of this method can be found in reference [Mistree, 1993].

MODELING OF HEAT SINK

A simplified heat sink parameter design model is constructed as an example. A heat sink on 30 X 30 mm ^{2} chips has been chosen. Configuration of the heat sink is shown below.

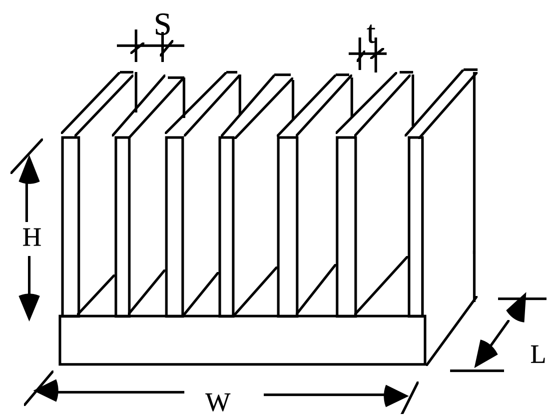

Figure 1. Prismatic Heat Sink

A heat sink is made of aluminum with a rectangular cross section having constant fin spacing. Also, forced convection with laminar flow is assumed for the system. Since forced convection is used, velocity of the air plays an important role. Dimensions of a heat sink are the other parameters which must be determined (See Figure 1). Design variables are velocity of the air(V), thickness of the fin(t), height of fin(H), length of fin(L) and gap between fins(S). The purpose is to find the design variables such that they minimize thermal resistance and cost. Additionally, heat loss limit, fin efficiency and laminar flow constraints must be satisfied.

To increase the heat rate to air in the vicinity of the heat sink, thermal resistance of the heat sink must be minimized. Lowering heat resistance enables more heat transfer, which is desirable. The heat transfer equation is shown below.

$$Q = \frac{\Delta T}{R} \quad \text{Heat transfer Equation}$$

As you can see, Q and R(resistance) are inversely proportional. Resistance of heat sink is defined as :

$$R = \frac{1}{1/R_f + 1/R_b}$$

where $R_b = \frac{1}{hA_b}$ and $R_f = \frac{1}{\sqrt{hPkA_c}\,\tanh(mH)}$

The second goal is related to the cost. Cost is assumed to be a linear function of velocity of air and volume of fin for simplicity.

Cost = C_1.V +C_2(t.H.L)

Constants in cost formulation are determined considering the dimensions of the sink. C1 is assumed to be $ 50 per m/s. Since C2 is proportional to the volume of a fin, C2 is equal to 1E6. $/m^3. In other words, 1 cm^3 volume is assumed as 1 $. Also constraints on the system must be achieved.

First, the heat loss limit is defined as :

$$h_l = 2k/ht$$

It is a measure whether the fin permits heat loss or not. h_l must be greater than 1 to utilize properly. Nusselt number is specified as 6.0 for fully developed flow.

$$Nu = \frac{hD_h}{k_f}$$

Second, fin efficiency must be greater than some specified value. In this case, it is 0.75. And, it is defined as :

$$\boxed{\eta = \frac{\tanh(mH)}{mH}} \quad \text{Fin efficiency}$$

where $\boxed{m = \sqrt{\frac{2h}{kt}}}$

The last constraint, flow, must be laminar. The Reynolds number defines the laminar region. In other words, Re must be less than the critical value of 2300. This constraint is represented as follows. The diameter Dh is the equivalent to the corresponding to gap. And ν is kinematic viscosity.

$$Re = \frac{V.Dh}{\nu} \le 2300 \quad \text{where } Dh = Dh(H,S)$$

After analyzing the system, the following Compromise DSP formulation was developed.

Compromise DSP Formulation

Given

Heat sink material: Aluminum
Type of fluid (air)

Find

Design variable	Notation
velocity of air	(V)
thickness of fin	(t)
height of fin	(H)
length of fin	(L)
gap between fins	(S)
deviation variables	(di)

Satisfy

Constraints

Heat lost limit :

$$\frac{2k}{ht} \ge 1$$

Fin efficiency :

$$\eta = \frac{\tanh(mH)}{mH} \ge 0.75$$

Reynolds No:

$$Re = \frac{V.Dh}{\nu} \le 2300$$

Goals

Fin resistance:

$$R = \frac{1}{1/R_f + 1/R_b}$$

$$\frac{R_g}{R} + d_1^- - d_1^+ = 1$$

Cost:

$$Cost = C_1.V + C_2(tHL)$$

$$\boxed{\frac{C_t}{Cost} + d_2^- - d_2^+ = 1}$$

Bounds on variables

$$0.02 \leq V(m/s) \leq 0.5$$
$$0.5 \leq t(mm) \leq 5$$
$$10 \leq H(mm) \leq 20$$
$$1 \leq S(mm) \leq 10$$
$$10 \leq L(mm) \leq 30$$

Minimize

$$\mathbf{Z1 = [}\ (d_1^+ + d_1^-), (d_2^+ + d_2^-)\ \mathbf{]}$$

This system consists of 5 design variables, 3 constraints and 2 goals. Three scenarios have been investigated to understand the sensitivity of the problem to the design variables, constraints and goal functions. Scenarios with different priority levels are shown respectively :

$$\mathbf{Z1 = [}\ (d_1^+ + d_1^-), (d_2^+ + d_2^-)\ \mathbf{]}$$
$$\mathbf{Z2 = [}\ (d_2^+ + d_2^-), (d_1^+ + d_1^-)\ \mathbf{]}$$
$$\mathbf{Z3 = [}\ (d_1^+ + d_1^- + d_2^+ + d_2^-)\ \mathbf{]}$$

In the first scenario, resistance is the first priority level while cost is the secondary level of priority. In the second scenario, cost is the first priority level while resistance is the secondary priority level. In the third scenario, both resistance and cost goals are the first level of priority.

The results obtained for design variables and deviation functions for different scenarios have been tabulated in Table 1 and Table 2.

SI	**Scenario1**	**Scenario2**	**Scenario3**
L(m)	0.01	0.01	0.01
T(m)	0.00141	0.00057	0.00141
V(m/s)	0.020127	0.020059	0.020058
H(m)	0.01572	0.01000	0.01572
S(m)	0.009993	0.009996	0.009995

Table 1 Design Variable Final Results

Dev F	Scenario 1	Scenario 2	Scenario 3
Level 1	0.0018784	0.0566151	0.06228419
Level 2	0.1859590	0.365248	----

Table 2 Deviation Functions

The trend of each design variable, deviation values and constraint violation values for scenario 1 are shown in Figures 2, 3 and 4.

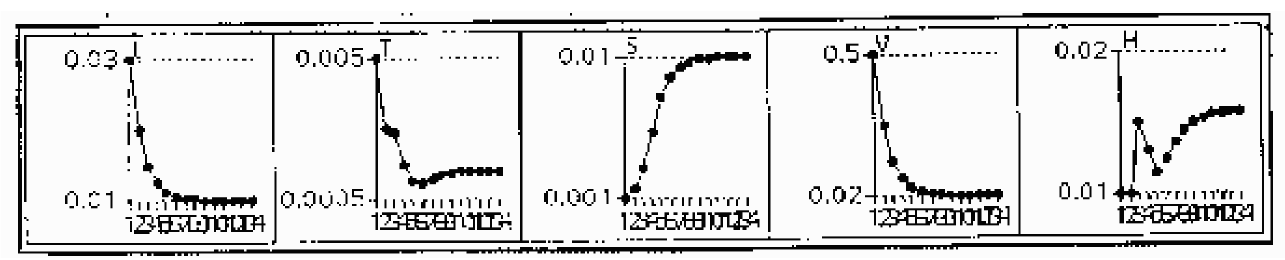

Figure 2 Iteration History of Design Variables for Scenario

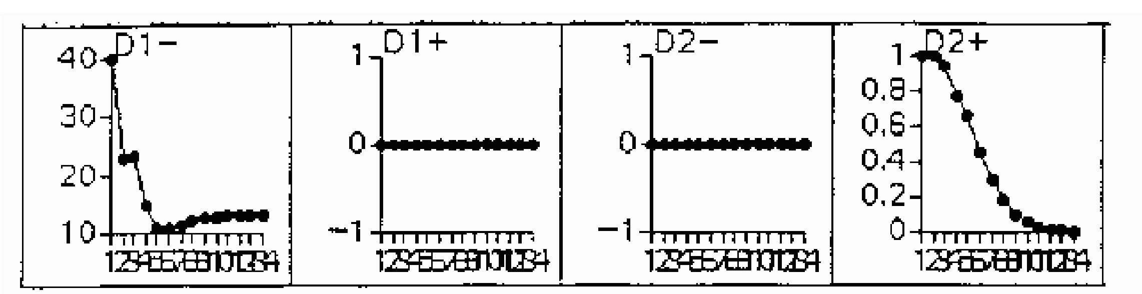

Figure 3 Deviation Variables for Scenario 1

The initial design point is the same for all scenarios. Design variables L, S and V converge smoothly. Meanwhile, variables T and H fluctuate at the beginning of the solution process, then they converge smoothly to the final value for Scenario 1 and Scenario 3. All the constraints remain feasible or inside the tolerable limit of constraint violation as seen in the figures. Constraint violation is zero in all three scenarios with the starting design vector given.

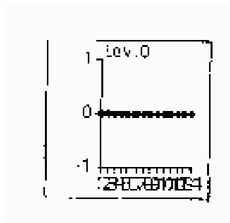

Figure 4 Constraint Violation For Scenario 1

The strategy to achieve the global optimum is to search from different initial points in design space and compare the results to verify that they are converging to the same optimum values. The results of the first initial design vector are shown in Table 1. Another set of initial design variables for scenario 1 has basically given the same results. The first and second initial starting variables are shown respectively:

X_{01} = { 0.03, 0.005, 0.01, 0.5, 0.01**}**

X_{02} = { 0.02, 0.001, 0.005, 0.15,0.015**}**

To assure accuracy of the results, more starting points can be used.

Two more scenarios have been searched. The purpose of that is mainly to focus on parameters, constraints and goals that are sensitive and dominant in the system. In the second scenario, the cost goal is the first priority; the resistance goal is the second priority. In the third one, each goal is of equal importance. Since the first and third ones produced the same results, cost is not a determining factor.

CONCLUSION

Parameter design of a heat sink on a chip sink has been done by facilitating optimization tools. Application of a multiple objectives approach during the parameter stage of the design can be helpful to the designer. The designer will have the flexibility to establish alternative scenarios and to evaluate and compare them. More elaborate models can be constructed for the heat sink and the model itself can be expanded including other components and electrical aspects of Electronic Packaging. Significance of this work is a multi objective approach to electronic packaging.

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